Review: Modeling Functions - 9/19/16

1 Models

A **mathematical model** attempts to express real-world problems in a mathematical way.

Some things that we use models for:

1. Population growth
2. Spread of disease
3. Demand for a product
4. International trade
5. Economic growth

To construct a mathematical model, we need to know about two types of variables: independent and dependent. The independent variable is the one that the experimenter manipulates, and the dependent one is the one that is being measured.

Example 1.0.1 *The population of deer is measured every day. The independent variable is the day, and the dependent one is the population of deer.*

Definition 1.0.2 *A model is **directly proportional** if as one value increases, the other increases at the same rate. Algebraically, this looks like $f(x) = kx$. The constant k is called the **constant of proportionality**.*

Example 1.0.3 *Suppose you are paid \$10 an hour. Then the amount that you are paid is directly proportional to the time that you work, with the constant of proportionality being 10. The formula for this model would be $f(x) = 10x$. Now if you got a signing bonus of \$5, then the amount you get paid would still be directly proportional to the time that you work, but now the formula is $f(x) = 10x + 5$.*

Definition 1.0.4 *A function is **inversely proportional** if the dependent variable is proportional to $\frac{1}{x}$. If one value increases, the other decreases, and vice versa*

Example 1.0.5 *When a shuttle launches, as the acceleration increases, the shuttle looses fuel and jettisons its rocket boosters, thereby losing mass. The force is constant, so the acceleration is inversely proportional to the mass, and the force is the proportionality constant. The formula for this is $a = F/m$.*

Practice Problems

1. We measure the number of people at Dartmouth who have the flu every day. What are the independent and dependent variables?

2. Suppose that every day, 5 people get the flu. What is the formula for the number of people with the flu over time?

3. Suppose we are driving on the highway and the distance we go is directly proportional to the amount of time we've driven. After 30 minutes, we've driven 35 miles. How far have we driven after an hour? What speed have we been going?

4. Suppose the time it takes to do a math problem is inversely proportional to the number of people working on it. One person can do a math problem in 30 minutes. How long does it take 3 people to do it?

2 Lagrange Interpolation

Definition 2.0.6 ***Lagrange interpolation*** *is a method by which, if we start with $n+1$ data points, we can build a polynomial of degree n that hits all the data points.*

Suppose we start with 2 data points, (x_1, y_1) and (x_2, y_2). Then our formula for Lagrange interpolation is:

$$f(x) = y_1 \frac{x - x_2}{x_1 - x_2} + y_2 \frac{x - x_1}{x_2 - x_1}.$$

Example 2.0.7 *You know that at the beginning of September you have \$145 and at the end you have \$58. Use Lagrange interpolation to estimate how much you have in the bank on September 19.*

Let's start by figuring out the data points: $(1, 145)$ *and* $(30, 58)$. *Then we plug these into the formula, so we get* $f(x) = 145\frac{x-30}{1-30} + 58\frac{x-1}{30-1} = -5(x-30) + 2(x-1) = -5x + 150 + 2x - 2 = -3x + 148$. *Now we plug in 19 to get* $f(19) = 91$. *So we predict that on September 19 you have \$91 in your bank account.*

Suppose we start with 3 data points, (x_1, y_1), (x_2, y_2), and (x_3, y_3). Then our formula for Lagrange interpolation is:

$$f(x) = y_1 \frac{(x - x_2)(x - x_3)}{(x_1 - x_2)(x_1 - x_3)} + y_2 \frac{(x - x_1)(x - x_3)}{(x_2 - x_1)(x_2 - x_3)} + y_3 \frac{(x - x_1)(x - x_2)}{(x_3 - x_1)(x_3 - x_2)}.$$

Example 2.0.8 *In the example above, suppose we know that on September 2, we still have \$145. Now our three data points are* $(1, 145)$, $(2, 145)$, *and* $(30, 58)$. *Our model will be*

$f(x) = 145\frac{(x-2)(x-30)}{(-1)(-29)} + 145\frac{(x-1)(x-30)}{(1)(-28)} + 58\frac{(x-1)(x-2)}{(29)(28)} = 5(x-2)(x-30) - \frac{145}{28}(x-1)(x-30) + \frac{1}{14}(x-1)(x-2)$. *Now if we want to predict how much money we had on September 19, we just plug in to get* $f(19) = 5(17)(-11) - \frac{145}{28}(18)(-11) + \frac{1}{14}(18)(17) \approx 112.21$. *So we predict that on September 19 you have \$112.21 in your bank account.*

Practice Problems

Suppose we're measuring the number of people who have the flu every day. On the first day, 1 person has the flu. On the 5th day of measuring, 12 people have the flu. On the 11th day of measuring, 30 people have the flu. Use Lagrange interpolation to find a polynomial that fits these three data points. Then predict how many people will have the flu on the 15th day.